A natural vierbein approach to Einstein's non-Euclidean line element in view of Ehrenfest's paradox

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From Riemann's non-Euclidean line element presented in 1854, the historical development led mathematically to general relativity theory up until 1916. Only later in 1928 Einstein introduced afterwards the tetrad formalism based on Cartan's concept of torsion, which then proved necessary for a general relativistic treatment of half-integer spin particles in quantum mechanics. In what follows, the other way round, a possibility is shown to start from a natural vierbein approach which applies to (substantial) physical reality in contrast to (mathematical) space and time themselves. This alternative proves compatible to Einstein's 1920 discussion in *"Geometrie und Erfahrung"* of Poincaré's corresponding 1902 ideas in "*La Science et l'Hypothèse"*. As well its basic presupposition is supported by a reanalysis of Ehrenfest's paradox. The latter problem induced Kaluza's 1910 introduction of non-Euclidean geometry into the framework of special relativity, before Einstein – based on the adaption of his original equivalence principle to local areas of space and time – transferred it to the gravitational field, leaving scope for an open question of interpretation.

1. Introduction

In physical application of Riemann's fundamental work [1], Einstein's non-Euclidean line element is the principal mathematical item of general relativity theory.[1)]

In his inaugural lecture of 1854 "*On the hypotheses which geometry is based on*"[2)] Riemann made *"... the presupposition that the lines have a length independently of the location ..."*[3)], and he stated *"... those features, by which space differs from other conceivable threefold extended quantities, can only be taken from experience"*[4)].

According to Riemann's work, these assumptions then shaped the view on non-Euclidean geometry[5)], and determined the further historical development up to Einstein's favored interpretation of his gravitational theory.

As early as of 1902, however, Poincaré wrote contradicting Riemann's historical statement in *L'Expérience et la Géométrie* [4] explicitly: *"... I have already tried to prove different times that the principles of geometry are no empirical facts, ..."*[6)]. He explains *"One may realize a material circle, then measure the diameter and the circumference, and try to see whether the relation of these two lengths equals π, what will have been done? One will have performed an experiment, not concerning the properties of space, but those of the material."*[7)]

On the other hand, obviously with regard to the title of his lecture, Riemann's epistemological argumentation culminates in *"These facts are (...) only of empirical certainty, they are hypotheses"*[8)], which words are clearly those of a mathematician, while a physicist might actually see it just the other way round[9)].

In clear contrast, Poincaré's definite conclusion is: *„The experiments consequently do not refer to space, but to the material objects."*[10)]

Anticipating an own section below, Einstein in his *Geometry and Experience* [6] states in unsurpassable clarity: *"As far as the sentences of mathematics refer to reality, they are not safe, and as far as they are safe, they do not refer to reality"*[11)], which insight should be realized as truly golden rule for mathematical physics at all.

Coming back to Riemann, his lecture later proceeds with *"If (...) one presupposes, like Euclid, a location-independent existence not only of the lines but also of the bodies, then it follows that the curvature measure is constant everywhere ..."*.[12)] – The other way round, Riemann's statement says that given a non-constant curvature – what in general relativity is the rule – rigid objects cannot exist independent from their respective location.

Though this conclusion does not contradict the possibility that under same conditions side by side – according to Ein-

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[1)] Also the abbreviations SRT, GR(T), RT are used in the usual meaning of S(pecial), G(eneral), R(elativity), T(heory).

[2)] *„Ueber die Hypothesen, welche der Geometrie zu Grunde liegen"* – Riemann [1], published in print 1868 only posthumously.

[3)] *„... unter der Voraussetzung, daß die Linien unabhängig von der Lage eine Länge besitzen ..."* – Riemann [1].

[4)] *„... diejenigen Eigenschaften, durch welche sich der Raum von anderen denkbaren dreifach ausgedehnten Größen unterscheidet, nur aus der Erfahrung entnommen werden können"* – Riemann [1].

[5)] Foundations have been developed by Gauß, Bolyai, Lobatschewski before; for an overview see literature e.g. [2], [3].

[6)] *„... j'ai déjà à diverses reprises cherché à montrer que les principes de la géométrie ne sont pas des faits expérimentaux (et ...)"* – Poincaré [4], Sect. V.1.

[7)] *„Qu'on réalise un cercle matériel, qu'on en mesure le rayon et la circonférence, et qu'on cherche à voir si le rapport de ces deux longueurs est égal à π, qu'aura-t-on fait? On aura fait une expérience, non sur les propriétés de l'espace, mais sur celles de la matière ..."* – Poincaré [4], Sect. V.2 (*"le rayon"* is translated to *"the diameter"* above).

[8)] *„Diese Tatsachen sind (...) nur von empirischer Gewißheit, sie sind Hypothesen"* – Riemann [1].

[9)] In contrast, the same year 1868 when Riemann's inaugural lecture of 1854 was published [1], Helmholtz came with an article titled "*Ueber die Thatsachen, die der Geometrie zum Grunde liegen*" [5], where in accordance with Riemann the fact is emphasized that e.g. a free relocatability of rigid objects would imply a constant curvature which may be of non-vanishing measure. As regards mathematical content his conclusions seem to reflect special aspects of Riemann's work.

[10)] *„Les expériences ont donc porté, non sur l'espace, mais sur les corps."* – Poincaré [4], Sect. V.7.

[11)] *„Insofern sich die Sätze der Mathematik auf die Wirklichkeit beziehen, sind sie nicht sicher, und insofern sie sicher sind, beziehen sie sich nicht auf die Wirklichkeit"* – Einstein in [6] „*Geometrie und Erfahrung*".

[12)] *„„Setzt man aber zweitens, wie Euklid, nicht bloß eine von der Lage unabhängige Existenz der Linien voraus, sondern auch der Körper voraus, so folgt, daß das Krümmungsmaß allenthalben konstant ist ..."* – Riemann [1].

stein's later criterion – proper standards equal each other always and everywhere (s. below), but it explicitly contradicts any possibility that then they stay absolutely unchanged (a simple counter example may be the common thermal expansion of e.g. the historical platinum-iridium meter sticks).

In addition to his statements above, Riemann's further explanation, *"That space is an unlimited threefold extended manifold, is a presupposition, which is applied at any conception of the outside world ..."* [13], will remain valid as long as any corresponding concept is based – due to exactly three spatial degrees of freedom – on a clear definition of physical space in the meaning of proven experience. [14]

Though several aspects as well as many more details may be left out in what follows – since it is impossible to cover the history of corresponding attempts, proposals, and arguments completely – it should make sense to discuss conceivable consequences of a new vierbein (tetrad) approach to general relativity in the context of Einstein's *Geometrie und Erfahrung* [6], Poincaré's *La Science et l'Hypothèse* [4], Riemann's *Über die Hypothesen...* [1], together with a reanalysis of Ehrenfest's fundamental paradox [7], even if only to check the validity of the Einstein's geometric interpretation of his wonderful equations

$$E_{ik} \equiv R_{ik} - \tfrac{1}{2} R\, g_{ik} = \kappa T_{ik} \ , \tag{1}$$

where E_{ik} is the Einstein tensor, R_{ik}, R are the Ricci tensor and its scalar, g_{ik} the fundamental tensor, and $\kappa = 8\pi G/c^4$ (with G the Newtonian gravitational constant, c the natural speed of light; in Einstein's 'extended' equations there would be an additional term Λg_{ik} with a cosmological constant on the right hand side).

Thereby several historical aspects of the development to general relativity and in particular to its interpretation are necessarily questioned, when now – though within scope – an alternative deduction of Einstein's non-Euclidean line element will be shown. While the underlying concept might doubt even the existence of two main problems arisen within traditional understanding, the same concept may prove without any loss of *reproducible* physical content.

A plausible choice between Einstein's favored kinematical interpretation and Poincaré's dynamical view will be considered and, with due respect, proposed after all.

[13] *„Dass der Raum eine unbegrenzte dreifach ausgedehnte Mannigfaltigkeit sei, ist eine Voraussetzung, welche bei jeder Auffassung der Aussenwelt angewandt wird ... "* – Riemann [1].

[14] The number of spatial dimensions is important because talking about a curvature of three-dimensional physical space would presuppose the existence of some non-curved at least four-dimensional physical space, too, where that curvature could take place. The other way round, it does not make sense to speak about curved lines where no straight lines exist. It will eventually prove unnecessary to ascribe any 'curvature' to three-dimensional space or time themselves. – One aspect is the display of local standards, the other would concern the universe. Furthermore, it may be kept in mind that all measured values – even if searching confirmation for up to e.g. eleven- and more-dimensional string theories at the LHC – are uniquely registrated as events in three-dimensional physical space.

2. A natural vierbein-approach

Einstein's theory of relativity is mathematically based in its first historical part on the fundamental tensor η_{ab} of SRT, where the indices $a, b\,.. = 1..4$ may refer to a coordinate system at rest [15], while indices $c', d'\,.., c'', d''\,.. = 1..4$ refer to systems in uniform motion. In its second historical part it is mathematically based on the fundamental tensor g_{ik} of GRT, where the indices $i, k\,.. = 1..4$ refer to any mathematically acceptable system S^i of arbitrary coordinates x^i describing continuously differentiable quantities.

Now the 'non-Euclidean' g_{ik} of GRT will be derived within flat space and with respect to a uniform time, both taken in the system S^a of *universal coordinates* x^a where matter and energy – homogeneously and isotropically distributed with respect to sufficiently large scales – are statistically at rest and, except for local deviations, the coordinate speed c^* of light, taken from the corresponding *proper* interval $d\sigma \equiv \sqrt{d\sigma^2} = 0$, equals its natural constant c.

Given two neighboring points $P(x^a)$ and $Q(x^a + dx^a)$ of this four-dimensional quasi-Euclidean [16] manifold S^a, their distance from an arbitrarily chosen origin as measured with systematically *affectable* physical rods and clocks [17], will be

$$\sigma^a_{\mathrm{P}} \ = \ x^a + \xi^a(x^a)\,, \tag{2}$$

$$\sigma^a_{\mathrm{Q}} \ = \ (x^a + dx^a) + \xi^a(x^a + dx^a)\,, \tag{3}$$

where the functions ξ^a are describing the respective deviations from the quasi-Euclidean values x^a due to physical deformation of the measuring standards. Now the second summand of σ^a_{Q} may be expanded according to

$$\xi^a\big(x^a + dx^a\big) \ = \ \xi^a\big(x^a\big) + \big[\partial_b \xi^a\big]\big(x^a\big)\, dx^b + \ldots \tag{4}$$

with $\partial_b \equiv \partial/\partial x^b$, hereafter the designator '(x^a)' will be omitted. The expansion (4) yields the 'properly' [18] measurable infinitesimal intervals

$$d\sigma^a \ \equiv \ \sigma^a_{\mathrm{Q}} - \sigma^a_{\mathrm{P}} \ = \ dx^a + \big(\partial_b \xi^a\big)\, dx^b + \ldots \tag{5}$$

between the two neighboring points Q and P. It is decisive to assign *by definition* an S^a quantity $\xi^a{}_b$ according to the second identity of the following expression

$$d\xi^a \ \equiv \ \big(\partial_b \xi^a\big)\, dx^b + \ldots \equiv \ \xi^a{}_b\, dx^b\,. \tag{6}$$

[15] At first, the term 'at rest' is only used in the sense of Einstein's 'rest system' („ruhendes System") in his fundamental work [8] on special relativity.

[16] The term 'quasi-Euclidean' implies uniform time.

[17] Here these measuring standards are presupposed to be used at rest, while otherwise the term 'affectable' will mean in general by gravitation and/or by motion.

[18] What is commonly called *proper* quantities means, these quantities are 'properly' measured (or measurable) using natural standards as e.g. spectral unit-sticks or e.g. atomic clocks.

Now *in general* it is

$$\xi^a{}_b \;\not\equiv\; \partial_b \xi^a \,, \tag{7}$$

where because of the '…'-symbol in (6), this non-identity (7) is valid except for *special* cases including SRT. According to (6), relation (5) may be written as

$$\mathrm{d}\sigma^a \;\equiv\; \mathrm{d}x^a + \mathrm{d}\xi^a \;\equiv\; e^a{}_b\,\mathrm{d}x^b, \tag{8}$$

with

$$e^a{}_b \;\equiv\; \partial_b x^a + \xi^a{}_b. \tag{9}$$

Here as well as in general, any upper contravariant index a,b.. may be lowered by definition again according to

$$\mathrm{d}\sigma_a \;\equiv\; \eta_{ab}\,\mathrm{d}\sigma^b. \tag{10}$$

Obviously the corresponding line element

$$\mathrm{d}\sigma^2 \equiv\; \mathrm{d}\sigma_a\,\mathrm{d}\sigma^a \tag{11}$$

follows from (10) and (8) by direct multiplication. With regard to a more general applicability, the total differential $\mathrm{d}x^b$ – corresponding to $\mathrm{d}x^a$ in (8) – may be replaced by

$$\mathrm{d}x^b \;=\; \left(\partial_{'c}x^b\right)\mathrm{d}x'^c \;\equiv\; \Lambda^b{}_{'c}\,\mathrm{d}x'^c, \tag{12}$$

which at first means only a Lorentz transformation from the universal frame S^a at rest to a system S'^a in uniform relative motion (the primes are placed in front of the indices respectively). Using correspondingly transformed local SRT tensors $\xi^a{}_{'c} \equiv \Lambda^b{}_{'c}\xi^a{}_b$ and $e^a{}_{'c} \equiv \Lambda^b{}_{'c}e^a{}_b$, relation (8) takes the form

$$\mathrm{d}\sigma^a \;=\; e^a{}_{'c}\,\mathrm{d}x'^c, \tag{13}$$

where

$$e^a{}_{'c} \;\equiv\; \Lambda^a{}_{'c} + \xi^a{}_{'c}\,. \tag{14}$$

In contrast to $\xi^a{}_{'c}$, here the additional contribution $\Lambda^a{}_{'c}$ – instead of $\delta^a{}_b$ in (9) – to the special 'deformation'-tensor $e^a{}_{'c}$ relates any properly measured intervals of the uniformly moving system to those measured in the universal frame. Then, lowering the upper index a in (13) according to (10) yields

$$\mathrm{d}\sigma_a \;=\; e_{a'd}\,\mathrm{d}x'^d\,. \tag{15}$$

Now expression (11) above yields from (15) and (13) by direct multiplication a corresponding line element in the form

$$\mathrm{d}\sigma^2 \;=\; g_{'c'd}\,\mathrm{d}x'^c\,\mathrm{d}x'^d\,, \tag{16}$$

where

$$g_{'c'd} \;\equiv\; e_{a'c}\,e^a{}_{'d}\,, \tag{17}$$

which in case of no gravitation – meaning $\xi^a{}_{'c} = \xi^a{}_b = 0$ – would reduce to the interim fundamental tensor $g^{\mathrm{SRT}}{}_{'c'd} = e^{\mathrm{SRT}}{}_{a'c}\,e^{\mathrm{SRT}\,a}{}_{'d} = \Lambda_{a'c}\Lambda^a{}_{'d} = \eta_{'c'd}$ of special relativity. – Finally with regard to a completely *general* applicability, the total differentials $\mathrm{d}x'^c$, $\mathrm{d}x'^d$ may be replaced according to

$$\mathrm{d}x'^c \;=\; \left(\partial_i x'^c\right)\mathrm{d}x^i\;, \tag{18}$$

which stands for another transformation of the S'^a coordinates to any arbitrary system S^i. This in turn means, relation (12) is modified to

$$\mathrm{d}x^b \;=\; \lambda^b{}_i\,\mathrm{d}x^i, \tag{19}$$

where the respective transformation coefficients $\lambda^b{}_i \;\equiv\; \partial_i x^b$ [$\equiv\; (\partial_i x'^c)\Lambda^b{}_{'c}$] represent the general coordinate transformation. In accordance with the correspondingly transformed 'half-tensors' $e^a{}_i \equiv (\partial_i x'^c)\,e^a{}_{'c}$ (s. Appendix c), relation (14) has to be replaced by the general expression

$$e^a{}_i \;\equiv\; \lambda^a{}_i + \xi^a{}_i\,, \tag{20}$$

where is $\xi^a{}_i \equiv \lambda^b{}_i\,\xi^a{}_b$. Now with respect to (18), relation (13) may be written

$$\mathrm{d}\sigma^a \;=\; e^a{}_k\,\mathrm{d}x^k\,, \tag{21}$$

as well as together with $e_{ai} = \eta_{ab}\,e^b{}_i$, relation (16) takes the form

$$\mathrm{d}\sigma^2 \;=\; g_{ik}\,\mathrm{d}x^i\,\mathrm{d}x^k \tag{22}$$

where, after all, as can be easily seen

$$g_{ik} \;\equiv\; e_{ai}\,e^a{}_k\;. \tag{23}$$

No peculiar property of three-dimensional space or of time is used in this derivation but merely a 'deformability' of physical rods and clocks by gravitation and motion according to the universal $e^a{}_b$ field itself.

This deduction yields not only Einstein's GR fundamental tensor g_{ik} which enables to effectively establish a non-Euclidean geometry of affected rods and clocks[19] – instead of the usually assumed one concerning 'space' and 'time' – but in particular, this immediately leads to the only appropriate form (23) to apply GRT to also half-integer spin particles described by the Dirac equation and its extensions.

[19] From 1918 up to posthumous editions (as e.g. one of 1963), in Einstein's „*Über die spezielle und die allgemeine Relativitätstheorie*“ it reads: *"We already know from earlier considerations that the behavior of measure-sticks and clocks is affected by the gravitational fields, i.e. by the distribution of matter."* – Einstein [9], third edition, 1918, see CPAE 6, Doc. 42 – („*Wir wissen bereits aus früheren Überlegungen, daß das Verhalten der Maßstäbe und Uhren durch die Gravitationsfelder, d. h. durch die Verteilung der Materie beeinflußt wird.*“)

Together with Einstein's equations (1) above, with his equivalence principle and his closely related 'geodesic' law of motion for test particles in an external gravitational field,

$$\delta\int \mathrm{d}\sigma \;=\; 0 \tag{24}$$

– which may be understood here the same way as other well-known principles of least action like e.g. that of Fermat, too – these relations are obviously a valid basis for an alternative approach to Einstein's theory of gravitation.

If it has been only for the non-Euclidean line element (22), (23) of GRT, however, one might have gone from (11) without an interim Lorentz transformation to a result $\mathrm{d}\sigma^2 = g_{cd}\,\mathrm{d}x^c\mathrm{d}x^d$ corresponding to (16), (17) by using the fundamental tensor η_{ab} within the universal frame S^a. Then – taking into account that, like any other scalar, $\mathrm{d}\sigma^2$ is naturally covariant with respect to arbitrary coordinate transformations – these expressions could be immediately generalized to (22), (23), in which case it would have been also sufficient, effectively to skip relations (12)-(18) at all. The full treatment above, however, may be more instructive including the 'strict' historical form of SRT as well as even its definite realistic approximation due to Einstein's equivalence principle, the latter applying only to freely falling local inertial frames up to those in the gravitational field of the large scale universe itself.

The other way round, in relations (18)-(23), any indices i,k.. associated to an arbitrary system S^i may be in return specialized to $'c,'d$.. associated to a system S'^a in uniform motion, as well as subsequently all latter indices of also (12)-(17) may be afterwards specialized to c,d.. associated to the preferred system S^a of universal coordinates[20] again.

A brief distinction of various cases including general coordinate systems without gravitation, too, is given with some comments in the Appendix.

After the introduction of teleparallelism and torsion by Cartan and others – for a review of the original historical development s. [10] and references therein – the general form (23) and its mathematical features are well-known as *vierbein* representation of the GRT fundamental tensor within the tetrad formalism framework as introduced in [11] by Einstein himself (see also e.g. [12] or [13]).

A particular feature immediately stated in [11] – where Einstein thought of a possible inclusion of the electromagnetic field into his teleparallelism extension (*„Fern-Parallelismus"*) of Riemannian geometry (s. also [14]) – is that in general 16 components of $e^a{}_i$ determine the 10 symmetric components of g_{ik} uniquely, while the other way round the latter (fundamental tensor) do not completely determine the first (tetrad). Any attempt to fully determine the tetrads, however, seems pointless. The existence of remaining 6 free parameters is necessary to allow for 4-dimensional rotations of particles, objects, and frames within the quasi-Euclidean universal frame without changing the non-Euclidean g_{ik} which describe gravitation. Therefore Einstein's search for tetrad field equations might finally concern a 'strength' of corresponding 4-dimensional vibrations, while the tetrads themselves may be regarded only 'potentials'.

In the conventional treatment of the tetrad formalism the expressions $\mathrm{d}\sigma^a = e^a{}_i\,\mathrm{d}x^i$ of (21) are assumed to be only sections of appropriately chosen coordinate axes at a given point X^i. Now it turns out that these quantities may be understood to be the proper intervals $\mathrm{d}\sigma^a$ – as measurable in principle by local natural clocks and rods[21] – of the non-affected quasi-Euclidean intervals $\mathrm{d}x^a$.

To find according to (8), (9), however, the actual expressions $\mathrm{d}\sigma^a$ in the field of e.g. a central mass at rest, would need the determination of local coordinates x^i merging into the universal ones x^a by application of a unique coordinate condition implying the behavior of the universal gravitational field's energy itself. Otherwise it would remain impossible to identify these coordinates[22], since they are irrelevant as long as only local 'pointlike' objects and their respective orbits are involved. Only neglecting gravitational energy, though, all of them as e.g. Schwarzschild's original coordinates, the isotropic, or harmonics ones might equally merge into the universal system.

Fifty years ago, Rosen [16] has pointed out an assumed link between his bi-metric formulation of GRT and the tetrad representation. In particular this work can further be used to identify that what is called 'space-time' – as based on the overstrained concept of proper quantities today – with only Einstein's gravitational potentials g_{ik} in infinite and non-singular quasi-Euclidean space and time.

Such a treatment might also offer a solution in principle of those two main problems of 20th century physics already mentioned above: (a) the alleged incompatibility of general relativity with quantum mechanics as well as (b) an assumed 'big bang' creation of the entire universe at all.

With regard to the covariance of Einstein's general relativity, it seems a natural claim, that relevant conclusions from his equations – like in the context of (a) or (b) concerning either 'space-time', or only the 'gravitational potentials' straightforwardly – should be independent of their respective interpretation. Hence, though all relations above are mathematically clear, their understanding may be subject to further discussion. Therefore, now the underlying basic aspects will be taken into explicit consideration, necessarily referring to the historical development again.

[20] Within the complete SRT framework, one 'preferred' inertial system may be arbitrarily chosen with respect to isotropy of the cosmological observations. Such a 'preferred' inertial system, however, cannot yet represent the universe, since its Riemann tensor – thus also matter and energy – would vanish there.

[21] These intervals $\mathrm{d}\sigma^a$ are respectively related to different neighboring points of space and time, where the difference may be spatial, temporal, or in general both. The extraction of the 3-dimensional spatial line element from a general 4-dimensional one is given by Landau & Lifschitz in §84, §89 of [13] (and elementarily addressed in [15], discussing Ehrenfest's paradox [7]).

[22] The assumed absence of a uniquely given quasi-Euclidean preferred frame to state the influence on proper quantities as measured by natural rods and clocks has been the essential reason for Weyl [2], § 33, to keep the literally geometric interpretation in spite of the mathematically equivalent alternative stated there as already by Einstein in [6] discussing Poincaré's insights, s. below. In another note it will be explicitly shown, why Weyl's argument of a missing appropriate reference frame may not apply.

3. Einstein's *"Geometry and Experience"*

Mathematics once started from practical demands, as in particular one branch to recover parcels of land by literally 'geometry' after floods.

Even today, if a physicist having measured a stationary though non-isothermal interior of a hall [23] reported, his 'geodesic experiments' yielded a locally variable non-Euclidean spatial line element, then – adhering to Euclid's old concept of mathematical space – everybody would immediately ascribe this result to a deformation of the measuring rods involved. The miracle is, that even such an originally unexpected geometry of deformable standards may be applied consistently.

In fundamental contrast, however, any conclusion from a non-Euclidean line element to a curvature of 'space' or 'time' themselves had to presuppose – as stated in extension of Riemann's concept [1] by Einstein himself [6] now – the existence of "rigid" measuring rods and non-affectable clocks with respect to gravitation or universal motion.

Just his own special relativity theory, though, proves the impossibility even of ideal rigid physical bodies and of ideal non-affectable physical clocks. Remarkably, it seems to have been Einstein again, who had demonstrated this fundamental consequence for the first time [17].

Later in the framework of general relativity, only a concept of *quasi-rigid*[24] standards really applies where, using Einstein's formulation in [6], the explicit definition may be: *"If once and someplace two distances are found equal, then they are equal always and everywhere"* (these "distances" are thought between corresponding pairs of marks on "two practically rigid objects", s. context of footnote 32 and in particular its last sentence below).

In the general relativity framework, of course, the necessity of non-Euclidean geometry is an unquestionable fact which, however, does not at all prove any physical properties of space or time themselves. In Einstein's 1921 article „*Geometry and Experience*" [6] he has explicitly accepted Poincaré's corresponding argumentation as an irrefutable valid alternative (equivalently in [18]).

Now following the statements in [6] step by step, Einstein wrote: *"In a reference system rotating relatively to an inertial system, the laws of arrangement for rigid bodies do not correspond to the rules of Euclidean geometry because of Lorentz contraction; so with acceptance of non-inertial systems as equitable systems, Euclidean geometry must be left."*[25] – Though the conclusion is right of course, the presupposition is not, because as will be shown in a reanalysis of Ehrenfest's paradox [7] below, Lorentz contracted "rigid bodies" in general do not exist.[26]

Einstein's statement is therefore misleading in this form, since even proper quasi-rigid spectral unit-sticks, placed peripherally around a rotating ring, are shown below to be *objectively* shorter in motion than those at rest.

If, on the other hand, the word "rigid" was rightly replaced by Einstein's term "practically rigid", then the concluded non-Euclidean geometry does not at all longer apply to space and time themselves, but rather to affectable rods and clocks instead.[27]

After Einstein's explicit acknowledgment of *"... the sharp witted and deep Poincaré"*[28], he wrote: *"Why is the suggestive equivalence of the practically rigid body of experience and the body of geometry rejected by Poincaré and other researchers? Simply because the real solid bodies of*

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[23] Corresponding temperature models have been already discussed in principle by Poincaré 1902 [4] as well as by Weyl [2] as already mentioned in the previous footnote, who nevertheless adhered to the interpretation in the sense of a 'curved' three-dimensional space, because it seemed impossible to determine a local deformation of rods without reference to a preferred frame. While this consideration remains valid as long as only GRT tensor quantities are involved, the situation will change essentially, when also the energy 'pseudo'-tensor (Rosen's bi-metric tensor) has to be taken into account (the corresponding problematic topic of gravitational waves is intended to be addressed in another note).

[24] To avoid any misunderstanding, in the paper at hand – except for literal citations – the terms 'quasi-rigid' (= 'proper', though clearly distinguished from simply 'rigid') will be occasionally applied instead of Einstein's rather ambiguous 'practically rigid' as used in original quotations. Correspondingly, all objects like in particular spectral unit-sticks may be named 'quasi-rigid' or only 'pr oper' again.

[25] *„In einem relativ zu einem Inertialsystem rotierenden Bezugssystem entsprechen die Lagerungsgesetze starrer Körper wegen der Lorentz Kontraktion nicht den Regeln der euklidischen Geometrie; also muss bei der Zulassung von Nicht-Inertialsystemen als gleichberechtigten Systemen die euklidische Geometrie verlassen werden."* – Einstein [6].

[26] In [8], Einstein mentioned a *"rigid sphere"* as a *"body, which examined at rest is of sphericity"* („starre Kugel" = „Körper, welcher ruhend untersucht Kugelgestalt besitzt"). In Einstein [19] the concept 'rigid' is restricted to *"rigid objects without acceleration and equally moved (i.e. relatively at rest to each other)"* – [*„... beschleunigungsfrei und gleich bewegter (d.h. relativ zueinander ruhender) starrer Körper"*].

[27] In this context Einstein's golden rule, quoted in the introduction, would also contradict e.g. Hilbert's 1916 statement in [20] *"... relativity theory (...) shows (...), that geometry and physics as one science are founded on common basis"* – [*„... die Relativitätstheorie (...) lehrt (...), daß Geometrie und Physik (...) als e i n e Wissenschaft auf gemeinsamer Grundlage ruhen"* (emphasis by Hilbert)]. – See s. also the CPAE 8 editors' note 8 following Doc. 278, a letter of 23 XI.16 to Weyl, where Einstein – qualifying Hilbert's ansatz as naïve („kindlich") – wrote that Hilbert's *"... 'axiomatic' method can be of little use"* – [*„... die ‚axiomatische' Methode kann dabei wenig nützen"*].

[28] *„... der scharfsinnige und tiefe Poincaré"* – Einstein [6]. Remarkably, by occasion of a fictive conversation in [18]'s *"Reply to Criticism"* discussing the same problem, Einstein once more explicitly acknowledged his *"respect of the [present] writer for Poincaré's superiority as thinker and author"* (square brackets in the English edition, in German: *„Respekt des Schreibenden vor Poincarés Überlegenheit als Denker und Schriftsteller"*), while in his writings – as pointed out in e.g. Pais [21] – Einstein rarely mentioned Poincaré at all. On the other hand, in a letter of 27.IX.22 to Zschimmer he wrote *"For myself, I take the view (...) also supported by Helmholtz and Poincaré e.g. ..." („Ich selbst stehe auf dem ersteren, z.B. auch von Helmholtz und Poincaré vertretenen Standpunkt ...")*, CPAE 13, Doc. 374 (the latter statement has also been cited by Hentschel in his introduction to a corresponding Sect. 4.5 of [3], where Poincaré's "Konventionalismus" is explicitly contrasted to other philosophical interpretations).

nature are not rigid at a more exact inspection because their geometric behavior, i.e. their possibilities of relative arrangement depend on temperature, outer forces, and so on."[29)]

Evidently, the different concepts of 'rigid' and 'practically rigid' are mixed up here, in that Einstein's 'practically rigid' – though implying possible influences of "outer forces" – tacitly excludes influences of gravitation or universal motion. Just in contrast, however, what is called 'quasi-rigid' in the paper at hand, applies consequently to *all* possibilities of dependence instead.

Thus from the beginning, seen from a logical aspect, Einstein arbitrarily excludes any dependence on gravitation or universal motion in addition to that of "outer forces" or temperature.[30)]

His concept of *"practically rigid"* („praktisch starr"), however, seems effectively confusing in this context since the condition above is also fulfilled by e.g. definitely non-rigid material meter-sticks proving congruent at various places in spite of different local temperatures between any two locations, if only both are side by side.

In fact, an "*equivalence of the practically rigid object of experience and the object of geometry"* has never been rejected by Poincaré, quite the contrary. Among many other subjects in [4], Poincaré discussed a temperature model, well conscious of the fact that all measuring sticks would fulfill an equivalent of Einstein's later condition above, if they only shared the same coefficient of expansion.

Einstein continues: *"... geometry (G) says nothing about the behavior of real things, but only geometry together with the concept (P) of the physical laws. (...) Sub specie aeterni, in my opinion Poincaré is right with this understanding ..."*[31)] – If anything might be considered 'sub specie aeterni' then what should this mean if not universal scales instead of local ones?

Going on, it reads: "*What concerns furthermore the objection that there are no really rigid bodies in nature, and that therefore their assumed properties do not apply to physical reality at all, this objection is not at all as deep as one might think at a brief consideration. For it is not difficult to determine the physical state of a measuring body so exactly, that its behavior concerning the arrangement relative to other measuring bodies becomes sufficiently unique, to substitute it for the 'rigid' body. The statements on rigid bodies shall be applied to such measuring bodies.*

All practical geometry relies on a principle accessible to experience which we want bring to mind now. We want to name a distance the perfect example of two marks fixed at a practically rigid body. We consider two practically rigid bodies and at each of them a marked distance. These both distances shall be called 'equal to each other' if the marks of the one can be continuously brought to coincidence with the marks of the other.

Now it is presupposed: ... "[32)]; here follows the definition of 'practically rigid' already quoted above which, however, is sufficient for two consistent possibilities of geometry, depending on whether there really exist absolutely rigid objects or not.

In particular, Einstein's proper local spectral-standards fulfill the condition of 'practically rigid' but not those of absolutely 'rigid', in that they *"are equal always and everywhere"* without necessarily remaining unchanged.

According to Poincaré, the whole historical mystery may be resolved by the following statement: Three-dimensional *non-Euclidean* geometry is the consistent mathematical tool to measure *Euclidean* space using standards of systematically variable length.[33)]

Here the term 'systematical' means that all sufficiently small measuring rods are uniformly changed together with all comparable objects depending on situation. The traditional laws of Euclidean geometry apply approximately to sufficiently small areas only.

A well-known model may be the non-isothermal hall mentioned above where, given a various distribution of temperature, the ordinary spatial line element would be $d\sigma_{\text{spatial}} = (1+k\vartheta)\, d\sigma_{\text{Euclid}}$ with k a common thermal expansion co-

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[29)] *„Warum wird von Poincaré und anderen Forschern die naheliegende Äquivalenz des praktisch-starren Körpers der Erfahrung und des Körpers der Geometrie abgelehnt? Einfach deshalb, weil die wirklichen festen Körper der Natur bei genauerer Betrachtung nicht starr sind, weil ihr geometrisches Verhalten, d.h. ihre relativen Lagerungsmöglichkeiten, von Temperatur, äußeren Kräften usw. abhängen."* – Einstein [6].

[30)] In *"Geometry as a Branch of Physics"* [22], Robertson considers – like Poincare [4] or Weyl [2] before (s. footnote 23) – another temperature example, there rightly emphasizing the universality of gravitation. This universality, however, does not at all exclude a corresponding deformation in addition to the 'non-universal' one caused due to e.g. various expansion coefficients by temperature, of course.

[31)] *„... die Geometrie (G) sagt nichts über das Verhalten der wirklichen Dinge aus, sondern nur die Geometrie zusammen mit dem Inbegriff (P) der physikalischen Gesetze. (...) Sub specie aeterni hat Poincaré mit dieser Auffassung nach meiner Meinung recht."* – Einstein [6].

[32)] *„Was ferner den Einwand angeht, daß es wirklich starre Körper in der Natur nicht gibt und daß also die von solchen behaupteten Eigenschaften gar nicht die physikalische Wirklichkeit betreffen, so ist er keineswegs so tiefgehend, wie man bei flüchtiger Betrachtung meinen möchte. Denn es fällt nicht schwer, den physikalischen Zustand eines Meßkörpers so genau festzulegen, daß sein Verhalten bezüglich der relativen Lagerung zu anderen Meßkörpern hinreichend eindeutig wird, so daß man ihn für den „starren" Körper substituieren darf. Auf solche Meßkörper sollen die Aussagen über starre Körper bezogen werden.*

Alle praktische Geometrie ruht auf einem der Erfahrung zugänglichen Grundsatz, den wir uns nun vergegenwärtigen wollen. Wir wollen den Inbegriff zweier auf einem praktisch starren Körper angebrachten Marken eine Strecke nennen. Wir denken uns zwei praktisch starre Körper und auf jedem eine Strecke markiert. Diese beiden Strecken sollen den „einander gleich" heißen, wenn die Marken der einen dauernd mit den Marken der anderen zur Koinzidenz gebracht werden können. Es wird nun vorausgesetzt:

Wenn zwei Strecken einmal und irgendwo als gleich befunden sind, so sind sie stets und überall gleich." – Einstein [6].

[33)] If Gauß had actually checked – as later Schwarzschild has alternatively done – the angular sum in a triangle by an optical measurement of mountain peaks and had found it different from 180°, then – according to Poincaré's [4] – a simple and irrefutable explanation would have been a previously unknown deflection of light rays (systematic, in addition to other effects).

efficient and ϑ a respective temperature, both for all local classical objects involved. The mere possibility to practice consistent geometry here, is – among other applications– the real advantage of that non-Euclidean tool originally developed as only a mathematical fiction.

This way to understand analogous inestimable achievements, is strongly supported by the fact, that even light does not at all propagate along the relativistic 'geodesics' of 3-dimensional space[34)]. Therefore a concept of light-ray geometry as also addressed by Einstein in his *Geometry and Experience* [6], fails due to the simple fact that not even ideal laser beams would allow to establish a unique non-Euclidean spatial geometry there.

Einstein wrote: *"The question whether the practical geometry of the world is Euclidean or not has a clear meaning and its answer can be only provided by experience. All physical measuring of length is practical geometry in this meaning, the geodesic and astronomical length measuring, too, if one recalls the experience that light propagates in straight line and, to be more precise, in straight line in the meaning of practical geometry."* [35)]

Though, on the one hand, experience clearly proves the necessity to apply non-Euclidean geometry, it does not at all prove this feature as a quality[36)] of physical space, but merely – as will be immediately shown in the reanalysis of Ehrenfest's paradox [7] – the affectability of rods and clocks by gravitation and motion with respect to a preferred frame as may be established by the universal gravitational field.

Only in case of arbitrary one- or two-dimensional subspaces which are described by non-Euclidean line elements, 'affectable' means standards curved *or* stressed *or* both (where stressed may mean stretched or compressed, without actual curvature). Concerning 3-dimensional physical space itself, however, it seems always sufficient to understand exclusively 'affectable' a property of variable length standards.

In particular, one plausible reason may be that there is evidently no fourth *spatial* dimension which necessarily had at least to contain physical Euclidean 'hyper-surfaces', if indeed, to allow an actual curved space to be compared with.[37)] Any 3-dimensional 'spatial curvature' can always be regarded describing the locally coinciding proper length of measuring sticks affected by up to 6 symmetric tensor components $g_{\alpha\beta}$. Thus their respective universal intervals are in general depending on orientation, too.

The traditional statement according to Poincaré, that Euclidean geometry may be the most straightforward approach seems evident from the fact that – as a special case of the more complicated non-Euclidean one – it does not presuppose any measurable properties of mathematical space and time, while on the other hand, all physical laws may always apply to real fields or objects only.

4. Reanalysis of Ehrenfest's paradox

The historical treatment of Ehrenfest's [7] rotating disk – subsequently to Born – in particular by Kaluza might have been decisive for the development of general relativistic geometrical concepts by Einstein at last.

According to special relativity theory [8], proper spectral *unit-sticks* are always displaying the same number of nodes in a standing light wave of a proper standard frequency between ideal mirrors at both ends of each stick. It is presupposed that the respective number of 'Lorentz-nodes' [38)] stays always the same, as long as any such stick can be regarded part of a local inertial frame at least for the time of an individual light signal going there and back between the mirrors. In addition may be also placed respectively a clock showing each interval of local proper light time, where the corresponding proper rest length will be uniquely related to by the natural constant c.

Einstein's applying assumption of equivalently such proper spectral unit-sticks with an observed congruence side by side always and everywhere if under same conditions, however, does not at all prove they are rigid.

In view of FitzGerald-Lorentz contraction discovered in 1889 [24] and 1892 [25] on the one hand, and seemingly superluminal velocities [17] on the other hand, the classical model of the rigid body had to be replaced. Regarding the first of these reasons, 1909 Born [26] introduced what then Ehrenfest called *"relative-rigid"* („relativ-starr"), who in [7] wrote that *"... for an observer at rest, at each instant each of its infinitesimal elements exhibits just that Lorentz contraction (...) which corresponds to the element center's instantaneous velocity."* [39)]

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34) If they did, then e.g. the famous deflection of light passing the sun ($R^i{}_{klm} \neq 0$) had to be only half its measured value. In case of no real gravitational but merely an acceleration field of e.g. the rotating disk ($R^i{}_{klm} = 0$), the situation proves this fundamental fact of no three-dimensional geodesics even more clearly (s. also the following footnote, where Einstein – in view of the preceding discussion of traditional geometry somewhat confusing – apparently meant the "straight lines" in 4-dimensional 'space-time'.

35) *„Die Frage, ob die praktische Geometrie der Welt eine euklidische sei oder nicht, hat einen deutlichen Sinn, und ihre Beantwortung kann nur durch die Erfahrung geliefert werden. Alle Längenmessung in der Physik ist praktische Geometrie in diesem Sinn, die geodätische und astronomische Längenmessung ebenfalls, wenn man den Erfahrungssatz zu Hilfe nimmt, daß sich das Licht in gerader Linie fortpflanzt, und zwar in gerader Linie im Sinne der praktischen Geometrie."* – Einstein [6].

36) *"According to general relativity theory space is endowed with physical qualities; so an ether exists in this meaning."* – Einstein [23] – *(„Nach der allgemeinen Relativitätstheorie ist der Raum mit physikalischen Qualitäten ausgestattet; es existiert also in diesem Sinne ein Äther".)*

37) Otherwise the reality of three-dimensional spatial curvature corresponds merely to a mathematical artefact of same kind as e.g. imaginary numbers. In this case, however, such a wording would be unnecessarily confusing (not only the public, though).

38) According to [15], of an internally standing light wave at rest neither identifiable nodes of the **E**-field nor of the **B**-field can be appropriately transformed to relative uniform motion, but only the nodes of $\mathbf{E} + [\mathbf{v}/c\times\mathbf{B}]$, what is called ‚Lorentz field strength' there.

39) *„ ... jedes seiner infinitesimalen Elemente in jedem Moment für einen ruhenden Beobachter gerade diejenige* L*ORENTZ-Kontraktion (gegenüber dem Ruhezustand) aufweist, welche der Momentangeschwindigkeit des Elementmittelpunktes entspricht"* – Ehrenfest in [7], while Born's notation still has been 'rigid' (*‚starr'*).

As well as the absolutely rigid body of Newtonian mechanics, also the 'relative-rigid' body of special relativity theory should remain dynamically unchangeable, but in contrast it should be *kinematically* deformable now. As is well-known, Ehrenfest effectively found that – in today's notation – the proper circumference of a disk in rotation cannot equal 2π times its proper radius.

The same day 29.IX.09 when Ehrenfest's note arrived at *Physikalische Zeitschrift*, Einstein wrote in a letter to Sommerfeld: *"The treatment of the uniformly rotating rigid body seems of great importance for me ...".* [40]

After two interim contributions of Born [28] and Herglotz [29], in a 1910 footnote of [30] Born commented, that Ehrenfest had shown *"... that a body at rest can never be set into uniform rotation"* [41], what he reported to have talked about with Einstein at the *"Naturforscherversammlung in Salzburg"*, which had took place at 19-25 September 1909 (the *"body at rest"* is obviously meant here to be a 'relative-rigid' one).

To explain the actual experience that material bodies can be set well into rotation, he refers to the atomic structure of matter and – not conscious of course of the historical irony – he stated *"There are no phenomena at all, being explained by rotations of electrons so far"* [42].

Before subsequently Planck [31] made a difference between Lorentz contraction in motion and any deformation during acceleration, as well as v. Laue gave a special relativistic proof [32] of the impossibility of extended rigid bodies at all, quasi backstage a brief article appeared.

As first shown there by Kaluza [33], the *spatial* line element on a rigidly rotating disk [43] is non-Euclidean [44], what solves contradictory conclusions of Ehrenfest's paradox [7] as far as one is tacitly presupposing physical objects to be ruled by Euclidean geometry.

Kaluza's mathematically pioneering note [45] – which objectively introduced non-Euclidean geometry into relativity theory about two years before Einstein's and Grossmann's *"Entwurf ..."* article [35] appeared – has been rediscovered by Stachel in [34], where the latter found the rigidly rotating disk *"... a 'missing link' in the chain (...) to the crucial idea that a nonflat metric was needed for a relativistic treatment of the gravitational field."*

In Janssen [36] is given a detailed discussion of various aspects of that problem with rotation, in particular pointing out that, given a superordinate inertial frame, the metric of a rotating system is *"not a solution of the* Entwurf *field equations"* (here *Entwurf* means [35] again). Later Janssen & Renn [37] have emphasized the key role of rotation in *"Untying the Knot"* again, where they have tracked and enlightened Einstein's historical way from the Zurich notebook to his final equations.

Stachel wrote: *"In spite of the importance he attached to the problem, and the intense discussion occasioned by Ehrenfest's paper, Einstein published nothing directly on the question during the next years"*, and *"... surprising (...) is the lack of mention of the rotating-disk problem in any of his papers on gravitational theory from 1907 through 1915."* [46]

In addition, the following considerations on Ehrenfest's famous paradox [7] might help to question the historical non-Euclidean concept which has effectively crystallized in spite of a possibly ongoing story. It will be shown here at first that Lorentz contracted rigid objects do not exist. At second, even Lorentz contracted relative-rigid objects can only exist locally in infinitesimal regions of extended bodies, where in this context 'locally' incudes free alignment except for a fixed location of its centers of mass. These considerations may be started from another striking version [47] now.

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[40] *„Die Behandlung des gleichförmig rotierenden starren Körpers erscheint mir von grosser Wichtigkeit ..."* – Einstein [27].

[41] *„... daß ein ruhender Körper niemals in gleichförmige Rotation gebracht werden kann; dieselbe Tatsache hatte ich schon mit Herrn A. Einstein auf der Naturforscherversammlung in Salzburg besprochen"* – Born in [30] – *Verhandlungen der Gesellschaft Deutscher Naturforscher und Ärzte. 81*. Versammlung zu *Salzburg*. 19-25 September 1909.

[42] *„Es gibt überhaupt keine Erscheinung, zu deren Erklärung bislang Rotationen der Elektronen herangezogen worden sind"* – Born 1910 in [30].

[43] Stachel: *"... a topic that has been the subject of extensive – and intensive – discussion from the early days of the special theory of relativity to the present"* – [34].

[44] According to Kaluza, the correspondingly resulting time-lag (*"Schlußfehler"*) of Einsteinian synchronization in particular on the rotating earth (explicitly addressed in [15]) does *not* contradict relativity theory. However, this statement is true as long as one observes that SRT laws do apply here only in local inertial frames of e.g. the rotating disk – and thus due to Einstein's transfer only in local inertial frames of real gravitational fields, as well as finally only in local inertial frames of the universe. Therefore, this conclusion may seriously question – not to say: contradicts – a *universal* expansion as understood today.

[45] One page and three half-lines.

[46] In spite of most knowledgeable clear arguments, I do not agree to John Stachel's 'hazarded guess' that the reason for this abstinence is due to the *"amazingly brief period – some time between mid-July and mid-October 1912 – when the problem played its role"*. Instead during the two years before, nobody might have been able – including Einstein himself and apparently all of his relativistic successors – to understand at first glance the physical consequences of Kaluza's solution. In my personal view, for Einstein this longer span of time between 1910 and 1912 possibly played the role of an intensive latent period, and this may be why, correspondingly, it would be no surprise that there is *"... no evidence that Einstein – or anyone else in the long history of the rotating-disk problem for that matter – was aware of the existence of Kaluza's work"*. Concerning evidence, however, the fact that Kaluza's brief article has not been explicitly cited by Einstein – just as little as the one of Ehrenfest – does not necessarily mean that he has not taken note of it. On the other hand, however, Kaluza has certainly not seen Ehrenfest's rotating disk even after its treatment by himself in any context of gravitation.

[47] Ehrenfest's original gedankenexperiment has been discussed in a German e-print [15]. It is also shown there that the principle of a constant speed of light only applies on same bidirectional two-way paths, which modification – compatible with Einstein's original formulation of SRT [8] – allows an *internal* synchronization of technical system clocks in uniformly rotating frames (to my knowledge for the first time), while the actual definition of the meter has to be correspondingly modified.

Regarding Einstein's synchronization principle given *"by definition"* in [8], it is true that sufficiently slowly separated clocks

4.1 Wheel stuck together and set into rotation

Consider a material wheel first at rest, stuck together of 314 peripherally orientated standard rods as rigid as possible without any variable thermal expansion, and respectively 100 radially orientated standard rods of same length $L_{\text{W-rest}}$ in any diameter of two spokes between each opposite peripheral splices (the number 100 may also stand for 1000, 10000, 100000… and correspondingly the number 314 for 3141, 31415, 314159… thus representing the leading digits of π up to any desired approximation). The splices may be each sufficiently small, technical problems are neglected here.

All processes will take place within an evacuated space station representing a freely falling inertial system, which for this special instant may be temporarily at rest. According to the definition in Section 2, 'at rest' actually means with respect to the quasi-Euclidean system of universal coordinates S^a determined by the statistical isotropy of all astrophysical observations over sufficiently large scales including redshift. A flat semitransparent screen is fixed within the isothermal station und thus at rest, too.

After the parallelly orientated wheel is set into a uniform rotation, its outlines are projected perpendicular to the screen. Then the shadow is measured, now using a second sort of standard rods at rest whose length L_U is chosen to fit all in all (not necessarily each to each) 100 times into the shadow's diameter, and accordingly 314 times around its circumference, because evidently the quotient is π for the shadow on the screen.

Now this second sort of standard rods is chosen by definition to be proper spectral *unit-sticks* (U) each with a clock displaying proper time and two mirrors at its ends, where its length $L_U = ½\, c\, T_U$ would always be exactly related to half the light time for a signal propagating there and back between both mirrors (with the natural constant c in general equaling the local *two-way* speed of light).

(α) Seemingly in accordance, the constant π is also found when the number 314 of the rotating wheel's peripheral stuck rods is divided by the number 100 of its radial stuck rods at any diametrically opposed pair of spokes.

(β) The 314 Lorentz-contracted peripheral stuck rods in rotation are each of same length $L_{\text{W-peri-rot}} = L_U$ of the 314 standard rods of the second sort at rest on the screen, since they continuously cover the wheel's *peripheral shadow* completely.[48)]

(γ) When, on the other hand, any one of the wheel's peripheral stuck rods in motion is measured with co-rotating spectral unit-sticks of identical construction to those above, transported to the wheel *after* set into rotation, then – according to SRT – due to Lorentz contraction each stuck rod will be found longer of length $L'_{\text{W-peri-rot}} = L_{\text{W-peri-rot}} / \sqrt{(1 - v^2/c^2)} > L_{\text{W-peri-rot}}$ (the prime respectively indicates *measured by proper unit-sticks U' in motion*, where v is the rotational velocity). Therefore, when measured with co-rotating spectral unit-sticks, it will be found a quotient larger than π since, except for thinkable stationary deflection, in both cases the length of any diameter – perpendicular to the direction of its motion – is the same.

If therefore a complete set of spectral unit-sticks were afterwards fixed with their centers – otherwise freely rotatable – at each splice on the rotating wheel, then the gaps between these unit-sticks would immediately prove the existence of that non-Euclidean line element on Ehrenfest's rotating disk [7] once found by Kaluza [33], also taken into account by Einstein [39], and much later explicitly discussed in Landau & Lifschitz [13].

The corresponding consequence is that – since otherwise (α) contradicts (γ) – there cannot exist any however-rigid standard rods maintaining an assumed 'proper'-ty after stuck together.

4.2 Rotating ring

Since this first step already shows rotation to imply deformability of composed physical objects like e.g. wheels, now it will be proved in a second step that any freely transportable proper unit-sticks are necessarily affected themselves, too.

To this end, the thought-experiment can still further be sharpened by focusing on the peripheral rotation only. Instead of the original disk or the wheel, now independently of how set into rotation consider a uniformly rotating ring instead, also parallelly orientated in front of that flat semitransparent screen introduced above. Here, the material of

always show middle in time reflection sending and receiving light signals within one and the same inertial frame. This means that clocks may be uniquely synchronized there by sufficiently slow transport without referring to any mathematical 'definition' instead. Though this concept would always apply locally, however, any such synchronization will fail in general.

Even on e.g. a rotating ring, Einstein's synchronization method as based on middle-in-time reflection – thus presupposing an unrealistic constant *one-way speed* of light – proves equivalent to a sufficiently slow clock transport, what may be easily verified by direct calculation. Though, in both cases it leads to Kaluza's 'time-lag' mentioned in footnote 44 above. In addition to the gravitational variability of its – tacitly assumed two-way – value stated by Einstein already in [19], [38], either in particular the one-way speed of light cannot be constant everywhere, or superordinate inertial frames have to be preferred in contrast to *general* relativity. In [15], respectively two different values for the 'coordinate' speed of light even in infinitesimal areas of the rotating disk are proved presupposing only one natural clock, where any questions of synchronization do not matter at all.

Only within *stationary* non-inertial systems, an alternative synchronization method may be necessarily established by another mathematical 'definition' to solve the unsatisfactory situation of different values of *proper* time at the same point in space, which again proves the inability of local SRT concepts globally to describe any non-inertial systems, even only one at a whole. Instead, the concept of a universal frame may be (re-)established, in that Einstein's GRT system coordinates are identified as respective representations of quasi-Euclidean space and time.

48) Although the number of rods of any shadow-spoke on the screen is also the same as in the material spoke itself, this – however – does not necessarily prove that each of the rotating spokes is of same length, since in contrary to the peripheral situation, there is due to different centrifugal forces for different stuck rods no symmetry within one spoke.

the ring may be chosen throughout the same as for all unit-sticks, too, without any variable thermal extension again.

The rotating ring's shadow is measured from behind the screen using spectral unit-sticks at rest as before. Evidently congruent, the rotating ring and its shadow are definitely of same circumference.

As commonly accepted, due to Lorentz contraction again, a larger value $C_U{}' = C_U / \sqrt{(1-v^2/c^2)} > C_U$ will be found measuring the same circumference of either the ring by co-rotating spectral unit-sticks U' in motion than of its congruent shadow by spectral unit-sticks U of identical construction at rest (thus straightforwardly reminding of Poincaré's foreseen "material circle" above).

This result proves that – independently of their respective common proper size – even Einstein's spectral unit-sticks, if not continuously side by side, will be in general of different length: Freely transportable *spectral unit-sticks in motion are objectively shorter than those at rest*, though with the same number of interference nodes each, and therefore cannot be regarded throughout rigid at all.

4.3 A complete transfer to the gravitational field

According to Einstein's 1913 transfer [35] of non-Euclidean geometry to the gravitational field – based on the later adaption of his original 1907 equivalence principle [19] now to *local* areas of space and time[49] – it is only consequent, finally to transfer the definitely observed deformability of spectral unit-sticks from the accelerated frame of the rotating disk to any gravitational field, too.

The reason why such a – possibly the only consistent – transfer has been originally left out, might be that the treatment of rotating frames has remained an unfinished story[50] even for the long time since then.

It might be objected that the deformability of spectral unit-sticks proved here, could only apply to mere 'acceleration fields' in contrast to real gravitational fields[51] where Riemannian curvature actually vanishes. But making such a distinction, then the one central claim of Einstein's interpretation is lost, that any effects of acceleration or gravitation should be – not only analogous, but – essentially the same.

4.4 Lorentz contraction no general vice-versa effect

Within special relativity, Lorentz contraction is understood a symmetric effect between objects of infinitely extended inertial frames. While the periphery of a rotating wheel has to be approximated by an infinite number of local inertial frames instead, there is only one coherent unique circumference $C_U{}'$ simply related to C_U by the usual formula of Lorentz contraction. This formula, however, is no longer applicable symmetrically.

In contrast, each of 314 additional spectral unit sticks, singly fixed in one of the 314 peripheral splices respectively, may be regarded temporarily to belong to an infinite inertial frame. Then these unit sticks will show a relative Lorentz contraction, though reciprocal only for infinitesimal intervals of space and time.

Regarding the whole of physical reality, however, there can be no doubt that the temporal relative Lorentz contraction of single rods fixed on the screen is not integrable with respect to a co-rotating observer, in clear contrast to that of the co-rotating rods with respect to an observer at rest.

A fictive overall symmetry of Lorentz contraction is broken by *general* relativity's necessity to apply *special* relativity to various local inertial frames, altogether covering the universe at any same time.

The view that the Lorentz contraction of the rotating wheel's peripheral ring cannot be a relative reciprocal vice-versa effect, is strongly supported by the evidence from Newton's bucket which shows – in particular given the plenty of various differently spinning objects – that here the wheel or these objects may have been set into rotation, but impossibly the one universe around.

4.5 No strict separation of kinematics from dynamics

After the conclusion from the wheel and the ring discussed above that non-Euclidean geometry implies deformability of proper objects and standards, now in addition, there is another necessary aspect concerning relativistic dynamics at all: it is impossible to adhere to Einstein's interpretation of SRT as pure kinematics if applied to the *whole* of *local* inertial frames. It is impossible to presuppose 'relative-rigid' rods stuck together whose (relative) deformation

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[49] After Abraham had claimed, see [40]-A/B, the validity of Lorentz transformations in infinitesimal areas of any gravitational field, Einstein later in [41]-A – followed by comments [41]-B/C on Abraham's polemics – tried to disprove this claim. In his first reply to Einstein – with hindsight today – obviously Abraham was right stating that in gravitational fields any infinitesimal Lorentz transformations will not be integrable, see [40]-c. Furthermore, Abraham seems to have helped to clarify the equivalence principle, before it has been understood and applied in Einstein's „*Die Grundlage der allgemeinen Relativitätstheorie*" [39] after all. Only nowadays, exactly this non-integrability – mathematically proved in [41]-A at first – is the reason why it seems inappropriate to apply local SRT *proper* concepts to an 'expansion' of universal distances (s. also [42]).

[50] In a letter to Einstein CPAE 8/A Doc. 225 of 6 June 1916, where Lorentz reported his success in deriving Einstein's gravitational equations from a variational principle (published in [43]), he erroneously stated that the nodes of a *standing* light wave generated at some point within a peripheral Lecher conduction fixed around the rotating earth would *move* along there, what contradicts the stationary Sagnac effect [44] discovered in 1913/14 and regularly observed in e.g. a fibre optic gyroscope today. In the framework of relativity theory this effect has been cleared up in detail (as well as various other observational facts) by Laue [45], s. also [15]. In his reply, CPAE 8/A Doc. 226 of 17 June 1916, Einstein did not object. He apparently accepted this assumption though with some modification "to a tiny percentage" (*„in winzigem Prozentsatz"*), what indicates that even after his concluding paper on general relativity of 1916, the processes on the rotating disk cannot yet have been completely understood at that time, though several concepts had crystallized before.

[51] There is the objective criterion that in real gravitational fields the Riemannian tensor is $R^i{}_{klm} \neq 0$, while in mere 'acceleration fields' with $R^i{}_{klm} = 0$, this tensor should vanish completely.

– as e.g. seen from the shadow above – should be always due to Lorentz contraction completely.[52].

The only escape from the dilemma of various contradictions seems to understand Lorentz contraction as one dynamical process among others which cannot be separated to explain what may continuously rule e.g. the behavior of that 314-sided rotating wheel above.

Kinematical and dynamical aspects have also been discussed in "*Drawing the line between kinematics and dynamics in special relativity"*, [47] with references therein, where Janssen defends the kinematical view of e.g. length contraction instead of Lorentz' and Poincaré's approach. There is rightly argued [47], though, that given a stable model at rest, *"Lorentz invariance guarantees that a contracted version of the same system in uniform motion will also be stable."*

The essential problem with Lorentz' and Poincaré's dynamical approach, however, might be that it seemed impossible to take into account local Lorentz invariance within that framework completely. As Einstein stated in 1907 [17], a consistent relativistic mechanics did not yet exist at all.[53]

With hindsight, concerning Janssen's statement above, the actual reason for a given stability of solid objects may be seen even more generally in the validity of Einstein's equivalence principle, where finite local inertial frames are governed by SRT-concepts, though temporarily and approximately only [s. also Appendix c) below].

Apparently Brown & Pooley [50] had argued in contrast to Janssen before: *"... that Minkowski space-time cannot serve as the deep structure within a 'constructive' version of the special theory of relativity."*[54]

Since a wheel or a ring can be set into rotation of course, Born's reference to the atomic structure of matter seems to support what is clear from the above: any rigid physical objects if was built from continuously distributed matter, would show fissures and cracks after set into rotation.

4.6 Euclidean space between non-Euclidean disks

The unnecessary concept to ascribe actual non-Euclidean geometry – here needed with respect to the rotating frame – to three-dimensional space instead to the physical objects involved, gets definitely clear regarding two or more observers, one of them doing her/his measurements on the surface of a first disk at rest, another one on the facing surface of a second disk in rotation, all objects situated within a superordinate common local inertial frame. Any light signals of both observers – which actually prove the respective geometry, either Euclidean or not – *are propagating through the same common space between* e.g. parallel adjacent disks.

Insisting to ascribe the non-Euclidean feature to space and time would force to ascribe different spaces and times to the arbitrarily close adjacent disks what then would make any consistent treatment of spatially separate different objects obsolete even within one ideal inertial frame, or – more realistic – the universe at all.

4.7 Hints to quantum mechanics

Kaluza's treatment of the rotating disk proved the impossibility to apply SRT 'proper' concepts of space and time except for comparably small local regions which are infinitesimal at least with respect to the universal frame.

On the one hand, Riemann's [1] up to Einstein's [6] presupposition of 'rigid' measuring standards whose lengths remain always the same in spite of different respective situations, is untenable as shown above.

On the other hand, however, as already Riemann [1] has explicitly stated as well as Einstein in his *Geometry and Experience* [6], too, it is only this presupposition of 'rigid' proper units which, sufficiently small, would allow to understand Riemann's 'curvature' to be a property of three-dimensional space.

Actually any non-Euclidean result of corresponding measurements shows that a rotating disk can neither be absolutely rigid, nor relative-rigid, nor quasi-rigid, nor a proper object at all. Thus Ehrenfest's paradox proves that a strict separation of relativistic kinematics from relativistic dynamics is impossible.

As Stachel 1989 pointed out in [34] with references therein, it has been the rigidly rotating disk, where the mathematical application of non-Euclidean geometry to Einstein's special theory once started from. Now it seems that a consequent treatment of Ehrenfest's paradox might even be more than that "missing link", but playing an analogue role for relativistic dynamics corresponding to that of the 'black body' for classical electrodynamics.

Going to atomic extensions of even (almost) pointlike rotating particles, then surprisingly, the impossible strict separation of kinematics from dynamics concluded from relativity theory above, has been found long ago within the framework of quantum mechanics: as a characteristic feature, in

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[52] Already [15] has shown that a strictly separated treatment of relativistic kinematics (Einstein, Minkowski) from relativistic dynamics (Lorentz, Poincaré) is definitely impossible. Even before, in contrast to all well-known *kinematical* paradoxa, a 1985 article [46] has introduced several seriously challenging *dynamical* paradoxa of special relativity. A suggestive hypothesis behind – 'forbidden' at that time – has been: *Relativistic mechanics consequently worked out […] will prove to be [extended] quantum mechanics.*

[53] Einstein wrote in [17] *„... weil wir ein vollständiges, dem Relativitätsprinzip entsprechendes Weltbild einstweilen nicht besitzen"* (s. previous footnote, too). According to a largely extended German version „Skizze einer offenen Theorie von Elektrodynamik, Gravitation, Quantenmechanik" of [48], also contained in [49], already Poincaré, like Lorentz before, kept the objective of consistent electrodynamics in sight. Maybe even the necessity of a forthcoming 'revolution' as a consequence of special relativity theory was on his mind, when he *"... might get timid in view of the hypotheses piling up, whose classification into a system seemed difficult up to the bound of the impossibility"* („ ... kopfscheu würde angesichts der sich auftürmenden Hypothesen, deren Einordnung in ein System ihm schwierig bis zur Grenze der Unmöglichkeit erschien"); the latter statement is reported in Hentschel's „Interpretationen" [3], p. 109, where a plenty of authors, arguments, and facets on relativity are compiled from 1905 to 1955.

[54] See also Brown's "*Physical relativity. Space-time structure from a dynamical perspective"* [51] with references therein.

Heisenberg's relation there are always combined kinematical uncertainties with dynamical ones (yielding Planck's constant respectively).

It is attracting attention that the dynamical paradoxa in [46] of special relativity (as already mentioned in footnote 52 above), are otherwise well known as those elementary prototypes of quantum mechanics, which describe previously pure kinematical problems of classical mechanics. Taken seriously, a complete treatment of any rotating object might even require an extended version of that theory, after all.

About two decades after the historical irony mentioned above that – according to [30] of 1910 – a relative-rigid body at rest could never be set into uniform rotation, just Born might consequently have argued the other way round, that a relative-rigid rotating electron could never be slowed down, thus conserving its spin. In this view, once might come even full circle.

5. Discussion

The mathematical foundations of non-Euclidean geometry have been developed by Gauß, Bolyai, Lobatschewski (s. footnote 5), and Riemann [1], then have been further extended by Christoffel, Ricci, Levi-Civita (s. [52] with references therein). As is well-known, the work of these authors laid the fundaments and widened the ground, before it has been definitely applied to physics by Einstein.

At first it seems one might be allowed to ascribe the proven feature of GRT's non-Euclidean geometry to space and time themselves. But a reanalysis of Ehrenfest's paradox now proves that the non-Euclidean line element – as in that special case first found by Kaluza [33] – has rather to be taken alternatively. In particular concerning three-dimensional space, it may be always sufficient to understand non-Euclidean geometry a systematic affectability of natural physical objects including respective measuring rods, too. The unexpected 'miracle' – based on those unique historical mathematical discoveries – is that it is possible to apply a consistent geometrical concept in spite of affectable rods.

Though, the problem of the rotating disk together with Einstein's equivalence principle, had led to the view that 'space' – in conjunction with 'time – may be curved by gravitation, since old Euclidean geometry would not work here.

While the latter half sentence irrefutably applies with respect to proper rods and clocks, however, the former seems unnecessarily concerning space [55] and time themselves.

Historically, Dirac's fundamental equation has inevitably required the completion of GRT – originally based on Riemannian geometry exclusively – by local tetrads for quantum mechanics. In particular Einstein himself [11], [14], [54] has effectively provided this extension, after Cartan and others, s. [10], had introduced such concepts before.

[55] Not to go unmentioned in this context the words of some brave writer: *"You will immediately stop calling space curved"* – a 1947 postcard to Einstein, s. Pais [53]. Though this has been found a strange demand, now in view of a natural vierbein approach within Euclidean space one might wonder what, besides historical reminiscences, actually forces to go on without Occam's razor again.

In general, all physical processes might be taken in quasi-Euclidean universal coordinates, where with respect to sufficiently large scales: (i) homogeneously distributed e.g. clusters of galaxies are statistically at rest, and (ii) the coordinate speed of light equals its natural constant [56]. Here it is where various contributions of gravitation and motion – the latter in form of translation and rotation – may determine the tetrads $e^a{}_i$ altogether now.

It is anything but coincidental that the mathematical description of spinning objects need mathematics going beyond pure Riemannian geometry, since it is even impossible accurately to define any angular momentum within the original GRT framework. In that the latter refers exclusively to what is called *proper* quantities it is quasi-dogmatically adhering to a pure geometric conception presupposing non-affectable standard rods. The actual reason for the failure straightforwardly to define GR angular momentum is that in the well-known SRT definition if transferred to GR, spatial non-proper coordinates will be necessarily involved, since otherwise there could not apply any non-local angular momentum conservation law, which seems safely confirmed in e.g. the context of indirect detection of gravitational waves.

Strictly speaking, already the validity of this law is sufficient to disprove the claim to absoluteness of the historical geometric approach, which thus evidently fails in reducing physics to exclusively Riemannian properties [1] of space and time, while the tetrad extension is going beyond.

Now – compatible to clear concepts of in particular Poincaré as well as to corresponding insights of Einstein himself– the non-Euclidean geometry of general relativity has been shown possibly to be nothing [57] but an appropriate [58] mathematical tool to deal with measuring standards which are systematically affected by gravitation and motion, the latter relative to the universal frame. [59]

Special relativity once started from Einstein's presupposition that there does not exist any physical background medi-

[56] In a hypothetical case of no matter and energy, however, universal coordinates might be understood as one particular Poincaré-group representation of infinite flat space and time, which always are mathematical fictions since in reality inertial SRT frames do not exist in cases other than those of local freely falling ones. Even without the conditions (i) and (ii) above and mathematically strictly adhering to SRT – as clearly known by Lorentz, Ehrenfest, Poincaré and others, in particular Sexl [55], too – one would be free to choose one preferred frame. Though this means a freedom to name it 'Newtonian' in a sense, it is called 'universal' here to avoid several inappropriate associations (according to author's treatment since [15])

[57] Except for trivial cases of curved ordinary surfaces only.

[58] With the perception of general relativity's understanding as effectively commonly accepted today – having hardened since 1916 in the blossoming of Einstein's gravitational theory with time – some valuable attempts at the further development of its interpretation apparently may have become stunted (s. also [56] in another context).

[59] At first, this statement simply applies to all processes in local gravitational fields. Special consequences, however, appear when trying to describe cosmological processes including some basic features of quantum mechanics, thus suggesting a reference to the universal frame (whose coordinates are otherwise also denoted as 'comoving' or 'conformal' ones, s. corresponding remarks in [42]).

um like e.g. 'ether'. Though this strict SRT concept has already been broken up with Einstein's 1920 *Äther und Relativitätstheorie* [23], its further development may have remained unfinished and – with regard to the cosmic microwave background and dark forms of matter and energy – even today seems still unclear in parts.

In spite of the fact, that ultimate quantum solutions of consistent Einstein equations [60], possibly extending first attempts in [48] or [49], may be found rarely if at all, a resignation in view of the assumed incompatibility of general relativity and quantum mechanics seems unjustified. As soon as one discards the strictly quasi-dogmatic interpretation of GR, several fundamental problems might change into chances, from particle physics up to that of 'black holes' which might prove supermassive non-luminous objects only. There is simply no need for geometric properties of space and time to recover the immense plenty of experimentally verified results derived from Einstein's equations.

Therefore it might be not Einstein's gravitational equations, but merely a false doctrine [61] of physical 'space-time' as mentioned above which quite obviously frustrates a consistent interplay of both fundamental theories.

A first concept of 'space-time' goes back to Minkowski [62] [59] after already Poincaré had showed in [64]-B the possibility to take relativistic space together with relativistic time a quasi-Euclidean manifold [63] before.

Nevertheless, it is inappropriate to claim, that the mathematical fiction of a non-Euclidean 'space-time' as something like a physical agent has been proved by the variety of the excellent confirmations of Einstein's equations [64] including those of relativistic motion, of course.

Given the number of spatial dimensions by three – corresponding to the physical degrees of freedom – it is always sufficient to calculate the influence of gravitational potential and universal motion on real physical objects including rods and clocks while – on the other hand – space and time might be regarded no physical objects themselves. [65]

Einstein's 'geodesic' law of gravitational motion might have been seen once a completely unexpected generalization of Galileo's law of inertia, which for its part had been understood a fundamental kinematical feature then. Any strict kinematical association, however, will be only conclusive as long as – the other way round – the law of inertia can not yet be reduced to gravitational motion in the universal potential at least approximately. [66]

6. Conclusion

It has been shown by the alternative tetrad deduction of the metric fundamental tensor above that – concerning spectral unit-sticks, whether "rigid" or "practically rigid" as originally assumed – in contrast to the consistent mathematical apparatus of general relativity theory, Einstein's favored historical *interpretation* of his equations might suffer from a contradiction to its own presuppositions (contradictio in adjecto). The reason is that non-affectable unit-sticks or non-affectable clocks do not exist.

Six years after his final formulation of GRT, Einstein in *Geometrie und Erfahrung* [6] explicitly agreed to Poincaré's 1902 understanding as explained in *La Science et l'Hypothèse'* [4] long before. Einstein wrote that *"sub specie aeterni"* Poincaré was right. Therefore – though effectively almost forgotten today – there is no imperative need for the still prevailing historical interpretation.

The peculiarity that the line element of Einstein's gravitational theory has been deduced now in a context of quasi-Euclidean SRT, is implying two insights. There exists one universal totality of only *local* inertial frames; and, except for mathematical fictions, it is sufficient to accept that, in general, any existing clocks, unit-sticks, as well as all other

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[60] A first step to a quantized quantum energy-momentum-stress tensor has been proposed in an approach based on a still preliminary though in its parts already consistent variational principle to a unified theory of electrodynamics, quantum mechanics, and gravitation {s. [48] with references therein; the collection [49] contains the largely extended German version mentioned in footnote 53}. While regarding the Klein-Gordon equation mathematical consistency seems achieved there, now a new variational principle including the Dirac equation for spin particles based on the vierbein approach above may be reported in another note.

[61] In contrast to the historical development, also today there might appear shouting Boeotians, after long time ago in particular *"… the sharp witted and deep Poincaré"* (Einstein's words, s. above) has shown that curved *three-dimensional physical* 'space' can be regarded to be nothing but actually an unnecessary mathematical fiction.

[62] Some time later, Einstein apparently stated *"Since the mathematicians have pounced upon the relativity theory, I don't understand it myself any more."* – [57], p. 33 – (*„Seit die Mathematiker über die Relativitätstheorie hergefallen sind, verstehe ich sie selbst nicht mehr"*). Though, with hindsight, this may sound like a joke, it seems that subsequently using Minkowski's mathematical concept – physically based on [8], [58] as well as in particular on Poincaré's [58]-B – Einstein's later interpretation of GRT might have been driven by Minkowski's pretentious 1908 'union' lecture [59]-B (somewhat continued by Hilbert up to at least [20]).

[63] In his original treatment Poincaré had shown the possibility to take special relativistic space together with special relativistic time as a quasi-Euclidean manifold multiplying the imaginary unity $\sqrt{-1}$ ($\equiv$ i) to the time coordinate respectively.

[64] Time after time, any community seems naturally inclined to infer from correct consequences of a theory on the correctness of their presuppositions. The geocentric conception of the world once has been thought to be true because e.g. the forecasts of eclipses for moon or sun as well as those for the positions of stars and planets have been exceptionally successful on this basis (apparently even better than in Copernicus' first heliocentric model after that).

[65] As a rule, what is called 'space-time' today, may be always translated to concepts like gravitational potential, the universal tetrad field, or the behavior of rods or clocks only.

[66] It may be remarked, that the SRT line element implying Galileo's law of inertia just corresponds exactly to that of the Stationary Universe Model (SUM) in [42] at each arbitrarily choosable reference point $t_R{}^*=0$ of universal time (in this context, any previously assumed uniform universal motions prove rather gravitational 'geodesic' approximations than a strict feature of fictive infinite inertial systems there).

objects of physical reality are actually affected by gravitation and universal motion.[67)]

In spite of the historical ambiguity of interpretation, there should be no longer any difference in physical processes depending on mere interpretation of the non-Euclidean line element, but unfortunately it seems. In particular this applies to the respective line element of cosmology or to that of quantum mechanics, where in both cases the problem may be in an inappropriate 'proper'-concept fixation.

Though it is widely believed that at least on Planck scales general relativity and quantum mechanics will prove incompatible, such a statement seems premature since Einstein's wonderful equations are not yet solved for a detailed quantum energy-momentum-stress tensor on the right hand side, but only for his phenomenological substitute – describing a perfect fluid – whose provisional nature once let him write of 'lumber instead of marble' [61].

It has been argued here that it may be unnecessary to accept a curvature of 'space' and 'time' themselves to draw all physically relevant – i.e. reproducible – conclusions. This should mean not only those conclusions, which have been experimentally confirmed so far, but also those which may be ever confirmed experimentally at all.

Just to demonstrate corresponding evidence, there has been given that simple derivation of Einstein's non-Euclidean line element from a natural vierbein approach without referring to any peculiar properties of three-dimensional space or of time. This viable alternative is understood to apply within a quasi-Euclidean universal frame to an overall tetrad field including gravitational potential and the well-known effects of motion at the same time. The universal frame is determined by a stationary statistical isotropy of all astrophysical observations over sufficiently large scales including redshift.[68)]

Thus, even though such a conclusion may not be a uniquely provable fact itself – since also mere mathematical fictions would be logically legitimate if only consistent – the applicability of *non-Euclidean geometry* within *Euclidean space* has been explicitly shown for Einstein's theory of gravitation in this note.

The closely related controversial question of Einstein's kinematical approach on the one hand, or Lorentz' and Poincaré's dynamical approach on the other hand, seems answered in favor for the latter by the reanalysis of Ehrenfest's paradox after all. As has been proved there, in contrast to a fictive assumption of non-affectable rigid standards to support 'curvature' as a property of space, actual Lorentz contraction effectively contradicts its reciprocal applicability or any uncritical transfer of corresponding SRT results beyond the plenty of local inertial frames in the universe.

Instead of a non-existing effect on 'space' and 'time' between any two differently rotating disks shown above, now the other way round – taken Einstein's equivalence principle seriously – the proved affectability of rods and clocks has to be consequently transferred from the rotating disk to the gravitational field.

The impossible strict separation of relativistic kinematics from relativistic dynamics necessarily concluded here, immediately points out a characteristic feature of quantum mechanics, where in Heisenberg's relation kinematical uncertainties are always combined with dynamical ones.

According to Einstein's proven statement *"... that the behavior of measure-sticks and clocks is affected by the gravitational fields ..."* ([9], s. above), it seems appropriate, to consider systematic affectability of rods and clocks the physical cause for any non-Euclidean line element of universal time and three-dimensional space fresh from a restarted interpretation if necessary[69)].

67) As stated in another context *"Lorentz versus Einstein"* [60], where Janssen wrote: "There is no need to worry about testability", also the alternative concept above may be subjected to as many additional experimental tests as the historical version itself (here concerning general relativity instead of the special one there).

68) According to the alternative concept of space and time, in [62], [63], [64]-A/B, [42] a Stationary Universe Model (SUM) has been shown to be the only arguable solution of Einstein's equations without cosmological constant. A self-contained presentation of that model is available as 2013 e-print arXiv:astro-ph/0312655v6 so far {there in relation (9) of the preliminary tetrad derivation, expression $\delta^a{}_i$ has to be replaced by $\lambda^a{}_i \equiv \partial_i x^a$}. – In the framework of that concept, a general principle of relativity actually means that freely falling local inertial systems allow for the existence of stable objects in spite of their accelerated motions, yet *locally* implying uniform velocities relative to each other.

69) Concluding this article it is author's concern to state thankfully that also what may appear strange at first glance, has been developed learning and trying to understand genius Einstein's theories, writings, and ideas, which besides all fundamental historical achievements are additionally embracing seminal questions within one unique scope towards an ongoing – possibly open end – story. In the meantime, after almost a century of invaluable proceeds, any reasonable revision of interpretation might contribute to let Einstein's actual equations – physically based on his unique equivalence principle – shine even more clearly.

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APPENDIX

It may be sufficient, to give a rough distinction of three main cases in this appendix. Any appropriately adapted relations are given here with “a”, “b”, or “c” added in the parentheses to each corresponding number.

a) Gravitation and motion in the universal frame

Since all equations of Einstein's general theory hold due to their covariance with respect to any arbitrary coordinate system, they hold in the universal frame S^a too. Keeping this system S^a without any coordinate transformation in (12) - (23), the resulting ‘non-Euclidean’ g_{ab} allow to describe the universal actuality in principle according to the tetrad-extended GRT with respect to this unique restframe here. Thus, taking relations (2) - (11) as given in the text, one may directly skip to

$$d\sigma^2_{\text{universal}} \;\equiv\; g^{\text{universal}}_{cd}\, dx^c\, dx^d\,, \qquad (22a)$$

$$g^{\text{universal}}_{cd} \;\equiv\; e_{ac}\, e^a{}_d \;\equiv\; \eta_{ab}\, e^a{}_c\, e^b{}_d\,, \qquad (23a)$$

with the universal tetrads $e^a{}_c$, $e^b{}_d$ as given in (8), (9). These results are in accordance with a circumstantial verification, of course, where in relations (18) - (23) of the text, any indices $i,k..$ associated to an arbitrary system S^i may be afterwards specialized to $'c,'d..$ associated to a system S'^a in uniform motion, as well as subsequently all latter indices in addition to those of (12) - (17) then may be afterwards specialized again, now to $c,d..$ associated to the preferred system S^a of universal coordinates once more.

It is easily realized that in this case any contribution otherwise found as $\Lambda^a{}_{'c}$ in (14) or $\lambda^a{}_i$ in (20) is reduced to $\delta^a{}_b$ again, thus showing that according to (8), (6), here the complete deviations $\xi^a{}_b dx^b = d\sigma^a - dx^a$ are caused by gravitation only. Therefore the function ξ^a may always describe the respective deviation of σ^a from the quasi-Euclidean value x^a due to gravitational deformation of the measuring standards.

Of course the universal frame contains the plenty of moving objects, which are assumed here to be affectable not only by gravitation, but also by motion. But the actual distance of two marks from any moving standard stick, left in the same instant of universal time from both ends, thus representing its length within the universal frame, is evidently not affected here.

b) Special relativity

As already stated, also special relativity is included in the treatment of Section 2 above. The group of all SRT coordinate systems may be regarded to result from an infinite number of Lorentz transformations of the one particular preferred universal frame S^a, called ‘at rest’. Mathematically this naturally implies that Lorentz transformations will also lead from any one system into any other within that group.

In case of *strict* SRT without gravitation, the function ξ^a is completely vanishing which would otherwise contribute to the tetrad field including a corresponding deformation of the measuring standards.

Among all possibilities implying $R^i{}_{klm} = 0$, the well-known SRT relations relevant in this context are found with respect to any appropriate system $S'(x'^b)$ considering such a fictive '$\xi^a{}_b = 0$' world. In the variables x'^b, x'^c,… the primes may indicate uniform universal motion (here additionally the indices $'b$, $'c$, ... themselves may indicate that by presupposition there is no effect of 'curvature' there). – The correspondingly adapted relations then explicitly read:

$$\sigma^a_{\mathrm{P}} \;=\; x^a\,, \tag{2b}$$

$$\sigma^a_{\mathrm{Q}} \;=\; x^a + \mathrm{d}x^a\,, \tag{3b}$$

$$\mathrm{d}\sigma^a \;\equiv\; \sigma^a_{\mathrm{Q}} - \sigma^a_{\mathrm{P}} \;=\; \mathrm{d}x^a\,, \tag{5b}$$

$$\mathrm{d}\sigma^a \;\equiv\; \mathrm{d}x^a \;\equiv\; \delta^a_b\,\mathrm{d}x^b \;\equiv\; e^a{}_b\,\mathrm{d}x^b\,, \tag{8b}$$

$$e^a{}_b \;\equiv\; \partial_b x^a\,, \tag{9b}$$

$$e^a{}_{'c} \;\equiv\; \Lambda^a{}_{'c}\;. \tag{14b}$$

$$\mathrm{d}\sigma^{\mathrm{SRT}}_a = \Lambda_{a'd}\,\mathrm{d}x'^d\,, \tag{15b}$$

$$\mathrm{d}\sigma^2_{\mathrm{SRT}} \;\equiv\; g^{\mathrm{SRT}}_{'c'd}\,\mathrm{d}x'^c\mathrm{d}x'^d\,, \tag{16b}$$

$$g^{\mathrm{SRT}}_{'c'd} \;\equiv\; \Lambda_{a'c}\,\Lambda^a{}_{'d} \;\equiv\; \eta_{'c'd}\;. \tag{17b}$$

While in particular relation (7) of the text is obsolete in this case of strictly no gravitation, and relations (18) - (23) may be simply omitted, the other relations remain unchanged. Then the quantities $\Lambda_{a'c}$, $\Lambda^a{}_{'d}$ are the usual constants of special relativity, of course.

According to (13), these quantities relate any quasi-Euclidean intervals $\mathrm{d}\sigma^a$ of the preferred universal frame to corresponding proper intervals $\mathrm{d}x^{'c}$ of inertial systems in uniform motion, leading from S^a to $\mathrm{S}^{'c}$ or the other way round.

Any non-zero deviations from the universal values $\mathrm{d}\sigma^a$ ($= \mathrm{d}x^a$ in this special case) would arise here from constant physical deformations of all objects including the measuring standards in uniform motion with respect to the universal frame. This is mathematically in full accordance to Einstein's relations of SRT.

In spite of this kind of material deformations – as primarily understood by Lorentz and Poincaré – the Riemann tensor $R^i{}_{klm}$ as calculated from (23) would still vanish here as presupposed above.

c) Tetrad-extended general relativity

Since a function of x^a and x^i at the same time, the gravitational deformation 'half-tensor' $\xi^a{}_i$ does not only refer to the one preferred quasi-Euclidean coordinate system S^a but to any additional set S^i of arbitrary coordinates x^i, x^k… as well[70]. This feature is clear indication to apply Rosen's bi-metric relativity (s. remarks in Sect. 2), now together with a unique fixation to the one universal frame S^a above.[71]

Then any transformation to a system S^i of arbitrary coordinates will yield $T_i{}^k$ and $t_i{}^k$ rightly. While by itself the first expression $T_i{}^k$ ($= 1/\kappa E_i{}^k$) will prove an ordinary GRT tensor, the second one $t_i{}^k{}_{\mathrm{GRT}}$ in its usual pseudo-tensor form will not. Only a mathematically correct transformation to a bi-tensor $t_i{}^k{}_{\mathrm{BRT}}$ including the replacement of ordinary derivatives by covariant derivatives with respect to the new system S^i (such a replacement of ordinary derivatives does not change actual GRT tensors). Hereafter $t_i{}^k$ will always stand for the bi-tensor $t_i{}^k{}_{\mathrm{BRT}}$ as a rule (BRT means Bi-metric Relativity Theory where the mathematically underlying metric of both is fixed to pseudo-Euclidean space and time, while the non-Euclidean one is related to affectable physical objects only).

In contrast to the concept of 'half'-tensors as e.g. $\xi^a{}_i$ in the text, where only one of two indices is originally related to the universal frame S^a, now the concept of 'bi'-tensors may apply to both indices of e.g. $t_i{}^k{}_{\mathrm{BRT}}$ respectively. Any additional contribution $\lambda^a{}_i \neq \delta^a{}_i$ in (20) to the full half-tensor $e^a{}_i$ – which according to (21) relates quasi-Euclidean universal intervals $\mathrm{d}\sigma^a$ to intervals $\mathrm{d}x^i$ of arbitrary coordinates – occurs due to some respective coordinate transformation.

Applying one *general* coordinate transformation from the start would be sufficient to get – of 'bi' – some arbitrary additional system S^i by replacing in (4) - (11) already any index "b" by "i". Then skipping (12) - (18), there are the corresponding relations of tetrad-extended GRT without an interim Lorentz transformation culminating in

$$\mathrm{d}\sigma^a_{\mathrm{GRT}} \;=\; e^a{}_i\,\mathrm{d}x^i\,, \tag{21c}$$

$$\mathrm{d}\sigma^2_{\mathrm{GRT}} \;\equiv\; g^{\mathrm{GRT}}_{ik}\,\mathrm{d}x^i\mathrm{d}x^k\,, \tag{22c}$$

$$g^{\mathrm{GRT}}_{ik} \;\equiv\; e_{ai}\,e^a{}_k \tag{23c}$$

at one go [the equivalent numbers (21) - (23) in Section 2 as well as the original relations (2) - (3) remain unchanged].

It is obvious that locally variable deviations of properly measured intervals $\mathrm{d}\sigma \equiv \sqrt{\mathrm{d}\sigma^2}$ from their (quasi-)Euclidean values inevitably cause non-vanishing values of what is called by the mathematical term 'curvature'. The corresponding treatment of general relativity is in the textbooks.

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[70] This means that according to [12] the expression $\xi^a{}_i$ – like $e^a{}_i$ (his $V^\alpha{}_\mu$) in (20) above – may be regarded as *one set of four* generally covariant 'coordinate' vectors. In the paper on hand, the notation 'half-tensor' is preferred since it is a covariant GRT tensor with regard to its index i, but only a contravariant SRT tensor with regard to its index a.

[71] Rosen's bi-metric relativity, which will be explicitly addressed in another note, may help in trying to reconcile Einstein's equations with several serious objections addressed in the text, going back to primarily Lorentz, Poincaré, Ehrenfest and some others against a quasi-dogmatic 'kinematical' interpretation not only of SRT but even of GRT in parts. Meanwhile the underlying principles have been also briefly elaborated in the wider context of straight SUM mentioned in footnote 68 above.

In contrast, briefly coming back to the reasoning of relation (6) in the text, it may be explicitly stated here once more that in all *special* cases where $\xi^a{}_i = \partial_i \xi^a$, relation (20) as a generalization of (9) may be written in the form $e^a{}_i = \partial f^a / \partial x^i$ where $f^a = x^a + \xi^a$. Then the g_{ik} in (23) do represent nothing but the fundamental tensor $\eta^{(f)}{}_{ik}$ after transformed[72] from SRT into a general coordinate system. This means that the Riemann tensor $R^i{}_{klm}$ together with its scalar R as calculated from (23) would still vanish again.

Even in the general case $R^i{}_{klm} \neq 0$ of gravitation, according to (6) with non-vanishing $\xi^a{}_b \neq \partial_b \xi^a$ it may be approximately written $\xi^a{}_i = \partial_i \xi^a + \ldots$, here implying $e^a{}_i = \partial_i f^a + \ldots$ where with respect to a system S'^a in temporarily uniform relative motion, the quantities $F^a{}_{'d} \equiv \partial_{'d} f^a(\mathrm{X}) = \Lambda^a{}_{'d}$ as local approximations of $e^a{}_i$ may be regarded constants of an otherwise variable $f^a{}_{'d} \equiv \partial_{'d} f^a$ at a given point X.

On these assumptions, relation (23) would yield the fundamental tensor $\eta_{'c'd}$ of special relativity. One has to take here into account, however, that any freely falling object, like e.g. a space station, does not represent always the same local inertial system (as has been occasionally shown in 'The self-restoring validity of SRT within local inertial frames' of [42] recently).

Taken together, the full treatment of Section 2 is not only easily including *local* SRT in gravitational fields, but it may include – at least in parts – the equivalence principle, too. Thus, three fundamental postulates of Einstein's relativity theory might culminate in one presupposition: *Using proper standards, except for cosmological effects any freely falling sufficiently small local frames are internally not affected by gravitation or motion.*

This presupposition may imply (a) a general principle of relativity including the special one; (b) a locally constant two-way speed of light; and (c) Einstein's equivalence principle, too.

While conclusions (a), (c) seem understood, also the requirement (b) of a local constancy of the two-way speed of light seems justified from the presupposition above because – in contrast to the infinitely extended fictive inertial systems of strict SRT – here, more realistically, only *local* inertial frames are presupposed to exist. Then, however, since the general principle of relativity might apply even to large local areas like e.g. the solar system as a whole, too, any experiment like that of Michelson and Morley [65] could not yield a shift of interference fringes other than "zero".

According to Einstein, the concept of 'general relativity' is obviously taking into account that there is no effect of free fall measurable in closed systems by physical means *internally*. In other words, all bodies or even extended frames are unaffected by free motion as long as they are sufficiently small compared with the scale of the gravitational field there.

This includes strict SRT as an idealization of unlimited inertial frames, where gravitation would not exist at all, and free fall thus would mean uniform motion relative to all other systems only.

In all cases of general non-Lorentz coordinate transformations where due to $\xi^a = 0$ gravitational deformation may vanish – though these results correspondingly apply within freely falling local inertial frames, too – any differentials $\mathrm{d}x^i$ of S^i do not represent directly measurable intervals of space and time as displayed using proper standards immediately.

According to footnote 21 of section 2, the extraction of the spatial line element from a general 4-dimensional one is given by Landau & Lifschitz in § 84, § 89 of [13].

Such a general coordinate transformation even implies the possibility to derive basic SRT results using the Galilean transformation. The simplest relativistic treatment of Ehrenfest's rigidly rotating disk already by Kaluza [33] makes a natural use of this chance {this has been explicitly shown in [15], for the correspondingly transformed expression of η_{ik} s. also [13], or in general e.g. the corresponding relation (3.2.7) of [12]}.

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[72] For the general transformation of η_{ik} see e.g. the corresponding relation (3.2.7) of [12].